

Solve

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 20 \end{bmatrix} \quad \left| \quad \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \neq A$$

 $C(A)$

$$-3(a + 2b = x)$$

$$3a + 5b = y$$

$$-b = y - 3x$$

$$b = 3x - y$$

$$a + 2(3x - y) = x$$

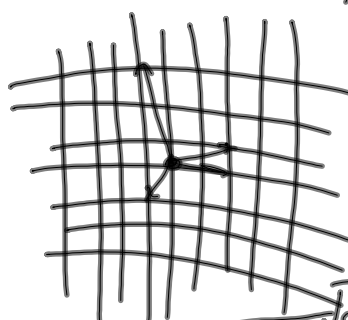
$$a = 2y - 5x$$

$$\begin{bmatrix} -5x + 2y \\ 3x - y \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



$$ax^2 + bx + c$$

$$\langle 1, x, x^2 \rangle$$



$$c\langle a, b \rangle = \langle ca, cb \rangle$$

$$c\langle 2, 0 \rangle = \langle 1, 0 \rangle$$

$$2\langle 1, 0 \rangle = \langle 2, 0 \rangle$$

$$\frac{1}{2}$$

$$a \in \mathbb{R} \Rightarrow \frac{1}{a} \in \mathbb{R}$$



$$1 + 1 = 0$$



$$2(\vec{v}) = 0$$

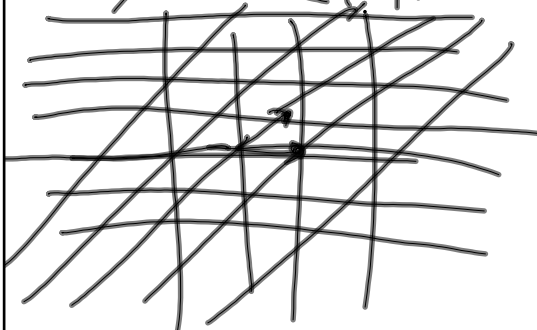
$$\vec{v} = 0$$

0	1
0 + 0 = 0	
0 + 1 = 1	
1 + 0 = 1	
1 + 1 = 0	

$$\langle a, b \rangle$$

$$a\langle 1, 0 \rangle + b\langle 0, 1 \rangle$$

$$\langle 1, 0 \rangle \quad \langle 1, 1 \rangle$$



Solve

$$\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

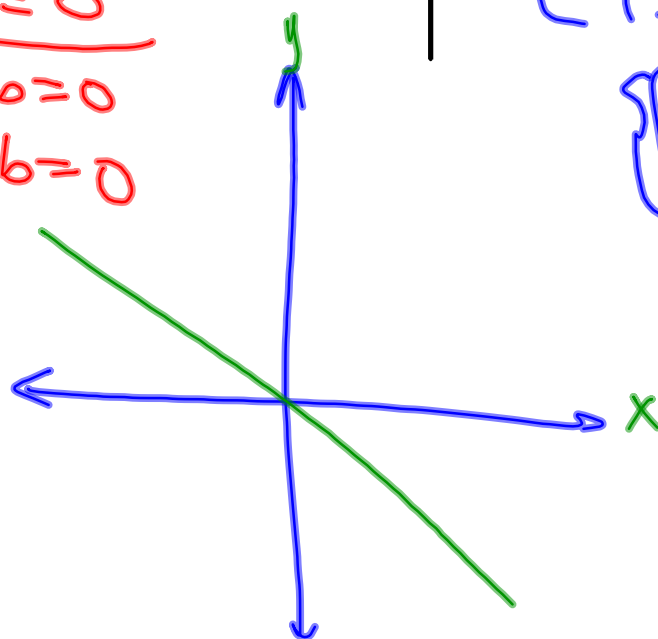
$$\rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2(a+3b=0) \rightarrow$$

$$2a+b=0$$

$$-3b=0$$

$$b=0$$



$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -2a \\ a \end{bmatrix}$$

$$\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \mid a = -2b, b \in \mathbb{R} \right\}$$

Nullspace

The *nullspace* of a matrix consists of all vectors x such that $Ax = 0$. It is denoted by $N(A)$. It is a subspace of \mathbf{R}^n , just as the column space was a subspace of \mathbf{R}^m .